3.7 Generalised dynamics

Lagrangian dynamics

Action	$S=\int_{t_1}^{t_2} L(\boldsymbol{q},\dot{\boldsymbol{q}},t)\,\mathrm{d}t$	(3.213)	S action ($\delta S=0$ for the motion) $\boldsymbol{q}$ generalised coordinates $\dot{\boldsymbol{q}}$ generalised velocities L Lagrangian t time
Euler–Lagrange equation	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=0$	(3.214)	
Lagrangian of particle in external field	$L=\frac{1}{2}mv^2-U(\boldsymbol{r},t)$ $=T-U$	(3.215) (3.216)	m mass $\boldsymbol{v}$ velocity $\boldsymbol{r}$ position vector U potential energy T kinetic energy
Relativistic Lagrangian of a charged particle	$L=-\frac{m_0c^2}{\gamma}-e(\phi-\boldsymbol{A}\cdot\boldsymbol{v})$	(3.217)	m_0 (rest) mass γ Lorentz factor $+e$ positive charge ϕ electric potential $\boldsymbol{A}$ magnetic vector potential
Generalised momenta	$p_i=\frac{\partial L}{\partial \dot{q}_i}$	(3.218)	p_i generalised momenta

Hamiltonian dynamics

Hamiltonian	$H=\sum_i p_i\dot{q}_i-L$	(3.219)	L Lagrangian p_i generalised momenta $\dot{q}_i$ generalised velocities
Hamilton's equations	$\dot{q}_i=\frac{\partial H}{\partial p_i};\qquad \dot{p}_i=-\frac{\partial H}{\partial q_i}$	(3.220)	H Hamiltonian q_i generalised coordinates
Hamiltonian of particle in external field	$H=\frac{1}{2}mv^2+U(\boldsymbol{r},t)$ $=T+U$	(3.221) (3.222)	v particle speed $\boldsymbol{r}$ position vector U potential energy T kinetic energy
Relativistic Hamiltonian of a charged particle	$H=(m_0^2c^4+\lvert\boldsymbol{p}-e\boldsymbol{A}\rvert^2c^2)^{1/2}+e\phi$	(3.223)	m_0 (rest) mass c speed of light $+e$ positive charge ϕ electric potential $\boldsymbol{A}$ vector potential
Poisson brackets	$[f,g]=\sum_i\left(\frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i}-\frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i}\right)$ $[q_i,g]=\frac{\partial g}{\partial p_i},\qquad [p_i,g]=-\frac{\partial g}{\partial q_i}$ $[H,g]=0\quad\text{if}\quad \frac{\partial g}{\partial t}=0,\quad \frac{\mathrm{d}g}{\mathrm{d}t}=0$	(3.224) (3.225) (3.226)	$\boldsymbol{p}$ particle momentum t time f,g arbitrary functions $[\cdot,\cdot]$ Poisson bracket (also see *Commutators* on page 26)
Hamilton–Jacobi equation	$\frac{\partial S}{\partial t}+H\left(q_i,\frac{\partial S}{\partial q_i},t\right)=0$	(3.227)	S action